Optimum Windward Performance of Sailing Craft

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A range of operating conditions is identified in which the hydrodynamic responses of hulls and sails appear adequately understood, and describable by a small number of parameters; viz., sailing to windward in light winds. The model developed for equilibrium of hull and sail forces includes profile and induced drag in both air and water, and emphasizes the conditions for optimum course selection and trim of sails. An analytic solution is found in terms of three or four dimensionless parameters describing the vessel, and an efficient numerical algorithm is outlined. Calculated results are given over the range of parameters encountered in conventional craft, and a numerical example is presented.

Nomenclature

A_s = total sail area
C_h, C_s = effective capture areas of hull and sail
D = drag force on sail
F, G = dimensionless functions of (x,τ,β), expressing conditions for equilibrium and optimum heading
L = lift force on sail
P_h, P_s = parasitic drag areas of hull and sail
q = dynamic pressure of water at V_S
Q = dynamic pressure of air at V_A
R = drag force on hull ("resistance")
S = lift force on hull ("side-force")
V_A = apparent wind speed
V_T = true wind speed
V_S = true vessel speed
x = "speed ratio" V_A/V_S
y = dimensionless performance criterion, $V_S \cos\gamma / V_T$
β = apparent wind angle
γ = angle of course from true wind direction
η = "efficiency" of hull or sail C/P
χ = "hull-to-sail capture ratio" C_h/C_s
ρ_a, ρ_w = densities of air and water
τ = dimensionless "trim variable" $L/2QC_s$

Subscripts

s = sail
h = hull
0 = starting value for iterative solution
x,τ,β = used as subscripts to signify partial derivatives of F, G, and y

I. Introduction

QUANTITATIVE analysis of the performance of sailing vessels through aerodynamic and hydrodynamic theory is a relatively recent development in a field where the empirical and intuitive approach to design has long been predominant and successful. In 1936 Davidson[1] laid down the principles by which tests of model hulls might be analyzed to predict the sailing speeds of a full-scale boat. Model tests have since played a prominent role in the design of many yachts, though the scaling of viscous and surface-tension effects on these models is well-known to be incorrect, and recent availability of full-scale towing data for comparison raises serious questions about the validity of model testing practice.[2]

The prominent papers on speed performance[3-6] have assumed that either the hull characteristics, or the sail characteristics, or both, are experimentally known for the particular yacht to be evaluated. Except for Crewe[3] most investigators of conventional yachts have avoided the question of optimum sail trim by use of the "Gimcrack coefficients," adopted as standard sail performance data by Davidson. However, use of these data involves some very restrictive assumptions about the planform and adjustment of sails, and does not admit that optimum trim of sails depends in an important way on the characteristics of the hull. While Crewe's analysis optimizes over two angles representing adjustment of sails, the two variables chosen do not form an adequate description of sail geometry, and they are intimately connected with experimental measurement of sail characteristics.

A fairly complete analytical model of sailing performance is possible today, and is being regarded by some as a useful tool for design and handicapping studies.[7-9] Because of lack of understanding of some aspects of the fluid mechanics – notably, the free-surface effects around the hull and the separated flow on the sails in some operating conditions—these models presently contain some tenuous assumptions that are hard to justify on the basis of the very limited amount of experimental data available. On the other hand, there appears to be an important regime of operating conditions for conventional craft in which the essential behavior of both hull and sails is well-understood, and can be represented by simple analytic descriptions. An investigation of optimum performance in this regime sheds light on the interaction between hull characteristics and sail trim and on the proper analytic approach, appropriate parameters, and the asymptotic solutions of a more complete theory.

II. Analytic Model

For purposes of a general treatment these conventions are adopted: the *hull* denotes the underwater portions of the vessel, and the *sail* refers collectively to all the parts of the vessel operating in the air (including the above-water portion of the body that is ordinarily named the "hull"). The basic assumptions of the model are:

1) Ambient conditions – water at rest, and air in uniform motion at relative speed V_T (the true wind).

2) Motion of the vessel – steady translation at speed V_S relative to the still water, at an angle γ from the true wind direction.

3) Hull characterized by constant parasitic drag area P_h, and constant effective aerodynamic span.

4) Sail characterized by constant parasitic drag area P_s, and constant effective aerodynamic span.

The conditions under which these assumptions appear appropriate are as follows:

1) Conventional craft sailing to windward in light winds. In the lower half of the speed range, deformations of the free surface around the hull are negligibly small, so the hull flow geometry is essentially independent of speed. Because of the size of typical craft and the low kinematic viscosity of water,

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Reynolds numbers are moderately high (e.g., speed 2 knots, keel chord 1 m, $Re \simeq 10^6$); the boundary layer is probably fully turbulent, thin and attached, at least on the hull lifting surfaces, and lift coefficients are low. Heeling of the vessel (trim angle about the longitudinal axis) does not markedly affect aerodynamic performance of the sail below about 20° of heel. Sail lift coefficients required for optimum sailing against the wind are often found to be somewhat below the maximum lifts as limited by stalling. It is estimated that a racing yacht is likely to spend 15-25% of its time operating in this regime.

2) Certain special craft in which heeling remains small and wave resistance is a negligible part of total hull forces – for example, some hydrofoil craft and some multihull designs. In these cases, the present analysis provides an upper bound on performance for the ideal limiting case where the harmful effects of heeling, wave resistance, and sail stall are completely avoided.

Dimensional analysis reveals that V_T is the only characteristic speed in the analytical model. Therefore V_S and the apparent air speed V_A are both proportional to V_T, and the angles between the velocity vectors are independent of V_T. Figure 1 shows the angular relationships between flow directions and forces in the air and water.

Equilibrium of forces parallel and perpendicular to V_S is expressed by

$$R = L \sin\beta - D \cos\beta \qquad (1)$$

$$S = L \cos\beta + D \sin\beta \qquad (2).$$

Let the hull be characterized, according to the assumptions, by the drag formula

$$R = P_h q + S^2/4C_h q \qquad (3)$$

where $q = \frac{1}{2}\rho_w V_S^2$, the water dynamic pressure; and C_h is an effective "capture area" for the hull. Defined in this way, the capture area of an elliptically loaded monoplane wing is $(\pi/4) \times \text{span}^2$. Equation (3) deliberately avoids the use of the arbitrary reference area needed for defining drag coefficient and aspect ratio, and describes the hull solely by the two areas P_h and C_h. (If a profile drag coefficient C_{Do} and an effective aspect ratio A_{eff} have been referred to the arbitrary area S_{ref}, P_h is identified with $C_{Do}S_{ref}$, and C_h with $(\pi/4)A_{eff}S_{ref}$.)

Similarly, let the sail be characterized by the drag formula

$$D = P_s Q + L^2/4C_s Q \qquad (4)$$

where $Q = \frac{1}{2}\rho_a V_A^2$, the air dynamic pressure; and P_s and C_s are parasitic and capture areas for the sail. (These two parameters may be similarly related to sail profile drag coefficient and effective aspect ratio referred to an arbitrary reference area.)

The view taken here, inspired by Tanner's lifting-line theory[10] and Milgram's lifting-surface theory[11,12] for sails, is that L can be varied continuously by all the geometric variables available in sail trimming – angle of attack, sail camber, and twist—so that C_s and P_s remain constant. We thus define a single dimensionless trim variable τ by

$$L = 2QC_s\tau \qquad (5)$$

to represent sail adjustment – essentially the lift coefficient of the entire rig, referred to an area of twice C_s. With this substitution, Eqs. (4) and (2) become

$$D = Q[P_s + C_s\tau^2] \qquad (6)$$

$$S = Q[2C_s\tau \cos\beta + (P_s + C_s\tau^2)\sin\beta] \qquad (7)$$

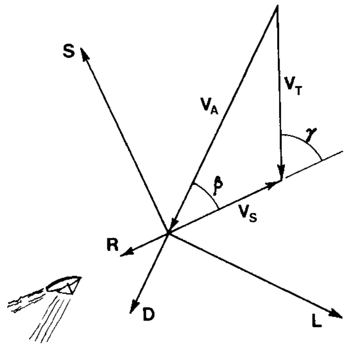

Fig. 1 Directions of force components in equilibrium.

L = lift, D = drag } "sail" S = lift, R = drag } "hull"

Now Eqs. (3)-(7) are substituted into the longitudinal equilibrium Eq. (1). With the further definitions:

"sail efficiency" $\eta_s = C_s/P_s$

"hull efficiency" $\eta_h = C_h/P_h$

"hull-to-sail capture ratio" $\kappa = \rho_w C_h / \rho_a C_s$

speed ratio $x = V_A/V_S$

the result is a nondimensional form of the equilibrium equation for steady sailing:

$$F(x,\tau,\beta) = 2\tau \sin\beta - (\eta_s^{-1} + \tau^2)\cos\beta - (\kappa/\eta_h x^2) - (x^2/\kappa)[\tau\cos\beta + \tfrac{1}{2}(\eta_s^{-1} + \tau^2)\sin\beta]^2 = 0 \qquad (8)$$

If Eq. (4) holds over a wide enough range of lift, the maximum lift-to-drag ratio (L/D) for the sail may be found from Eqs. (5) and (6) to be $\eta_s^{1/2}$, occurring at $\tau = \eta_s^{-1/2}$. Likewise, the maximum S/R for the hull described by Eq. (3) is $\eta_h^{1/2}$. However, for optimum performance of the vessel both hull and sail must in general be operated at lift-to-drag ratios below their respective maxima. Lift-to-drag ratios have been unduly emphasized in many past discussions of performance of yacht hulls and sails; consequently the "efficiencies" η_s and η_h are introduced here as more fundamental and characteristic quantities.

III. Extremal Problems

Given values for the trim variable τ, the apparent wind angle β, and the parameters η_s and η_h, Eq. (8) is quadratic in x^2/κ and can be solved explicitly for x. However, β and τ are control variables, the former being set by steering the craft and the latter by trimming the sails, and one or both will be chosen so as to optimize performance. First, for a given β it might be asked: what is the optimum value of τ, making $x = V_A/V_S$ a minimum? For any $(dx, d\tau, d\beta)$ such that Eq. (8) is satisfied,

$$dF = F_x dx + F_\tau d\tau + F_\beta d\beta = 0 \qquad (9)$$

where the subscripts denote partial derivatives. For β constant ($d\beta = 0$) and x a minimum ($dx = 0$), it is necessary that F_τ vanish:

$$F_\tau(x,\tau,\beta) = 2\sin\beta - 2\tau\cos\beta - 2(x^2/\kappa)[\tau\cos\beta + \tfrac{1}{2}(\eta_s^{-1} + \tau^2)\sin\beta](\cos\beta + \tau\sin\beta) = 0 \qquad (10)$$

The quantity x^2/κ can be eliminated between Eqs. (10) and (8), resulting in a quartic equation for τ; a solution in closed form is thus possible, though not computationally attractive.

The only parameters appearing in the quartic for τ are η_s and η_h (and of course β). This shows that for a given apparent wind direction the optimum sail trim depends on η_h as well as η_s, but not on κ. However, the resulting V_S/V_T depends on all three boat parameters. As shown in Fig. 2, the boat speed and direction relative to the true wind speed and direction are given in terms of x and β by

$$V_S/V_T = (1+x^2-2x\cos\beta)^{-1/2} \tag{11}$$

$$\cos\gamma = (x\cos\beta - 1)(1+x^2-2x\cos\beta)^{-1/2} \tag{12}$$

A more important extremal problem is to ask for the values of x, β and τ satisfying Eqs. (8) and (10) and giving the largest component of boat speed contrary to the true wind direction, $V_S\cos\gamma$, relative to V_T. The objective function to be maximized in this case is from Eqs. (11) and (12)

$$y(x,\beta) = V_S\cos\gamma/V_T = \frac{x\cos\beta - 1}{1+x^2-2x\cos\beta} \tag{13}$$

Thus

$$dy = y_\beta d\beta + y_x dx = 0 \tag{14}$$

Using Eq. (10) to define the optimum sail trim and eliminating $d\beta$ or dx between Eqs. (9) and (14), we find the three simultaneous equations for x,β, and τ are Eqs. (8), (10), and

$$G(x,\tau,\beta) \equiv F_x y_\beta - F_\beta y_x = 0 \tag{15}$$

The indicated derivatives can be taken without difficulty:

$$F_\beta = 2\tau\cos\beta - (\eta_s^{-1}+\tau^2)\sin\beta - 2(x^2/\kappa)[\tau\cos\beta + \tfrac{1}{2}(\eta_s^{-1}+\tau^2)\sin\beta][-\tau\sin\beta + \tfrac{1}{2}(\eta_s^{-1}+\tau^2)\cos\beta] \tag{16}$$

$$F_x = (2\kappa/\eta_h)x^{-3} - (2/\kappa)x[\tau\cos\beta + \tfrac{1}{2}(\eta_s^{-1}+\tau^2)\sin\beta]^2 \tag{17}$$

$$y_\beta = -x(x^2-1)\sin\beta/(1+x^2-2x\cos\beta)^2 \tag{18}$$

$$y_x = -[(1+x^2)\cos\beta - 2x]/(1+x^2-2x\cos\beta)^2 \tag{19}$$

However, the expression for G does not simplify much, and further algebraic efforts appear unrewarding.

IV. Numerical Solutions

For particular values of the parameters η_s,η_h, and κ, simultaneous solutions of Eqs. (8), (10), and (15) may be sought by numerical root-finding methods. The numerical analog of Newton-Raphson iteration[13] provides a simple algorithm that converges rapidly if the starting point is close enough to a root. The iteration is defined by

$$\begin{bmatrix} F_x & F_\tau & F_\beta \\ F_{\tau x} & F_{\tau\tau} & F_{\tau\beta} \\ G_x & G_\tau & G_\beta \end{bmatrix} \begin{bmatrix} x_{n+1}-x_n \\ \tau_{n+1}-\tau_n \\ \beta_{n+1}-\beta_n \end{bmatrix} = - \begin{bmatrix} F \\ F_\tau \\ G \end{bmatrix} \tag{20}$$

wherein F,G, and all their derivatives are evaluated at the point (x_n,τ_n,β_n). Exact expressions have been given in the preceding for F,G, and the first derivatives of F. They may likewise be obtained for the second derivatives required here; however, the one-sided numerical approximation by a secant, e.g., $F_{\tau x} \simeq [F_\tau(x+\delta,\tau,\beta) - F_\tau(x,\tau,\beta)]/\delta$ has worked satisfactorily in practice, using $\delta = 10^{-4}$ in a machine with 8-digit accuracy.

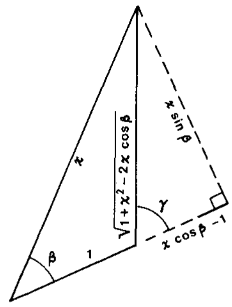

Fig. 2 Geometry of windward sailing, nondimensionalized by boat speed V_S.

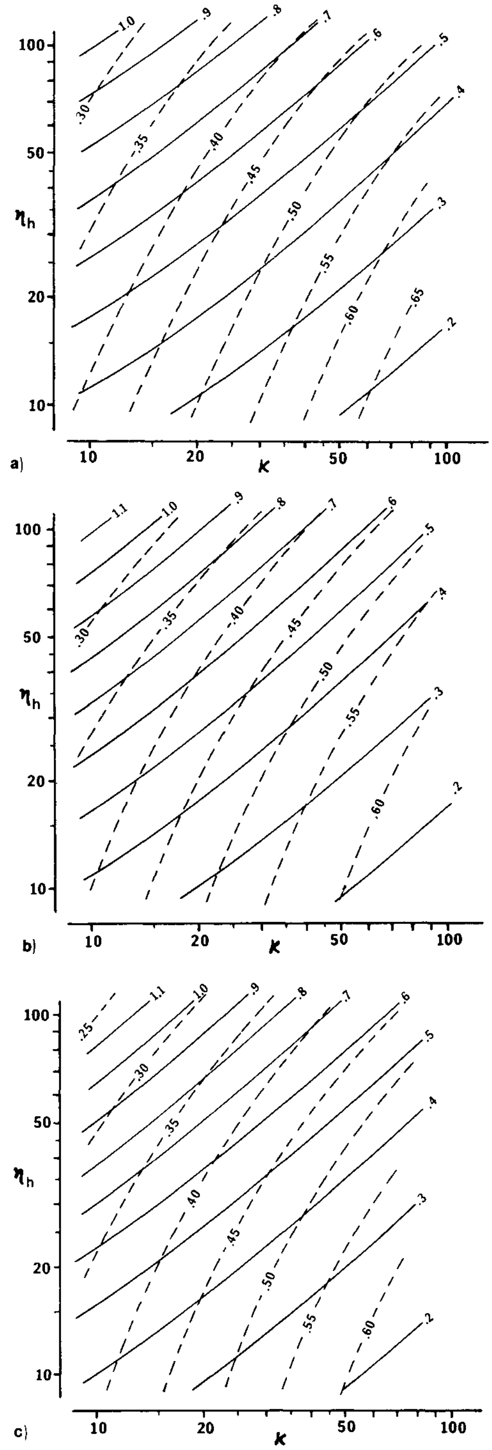

Fig. 3 Contours of the maximum value of the objective $y = V_S\cos\gamma/V_T$ (solid lines), and the optimum value of the trim variable τ (dashed lines) as functions of hull efficiency, sail-to-hull capture ratio, and sail efficiency. (a) Sail efficiency = 20. (b) Sail efficiency = 30. (c) Sail efficiency = 40.

In the absence of an approximate theory for obtaining good starting values (x,τ,β), it is safest to work away from a point (η_s,η_h,κ) for which the solution is known. The coefficients $\eta_s=30$, $\eta_h=30$, $\kappa=30$ are about average for racing craft and are used here as a basis for the following formula

$$\begin{bmatrix} x_o \\ \tau_o \\ \beta_o \end{bmatrix} = \begin{bmatrix} 2.196 \\ 0.471 \\ 0.517 \end{bmatrix} + \begin{bmatrix} +0.72 & +17.0 & -14.8 \\ +0.76 & +2.20 & -3.56 \\ +0.84 & +3.14 & -2.44 \end{bmatrix} \begin{bmatrix} \eta_s^{-1}-30^{-1} \\ \eta_h^{-1}-30^{-1} \\ \kappa^{-1}-30^{-1} \end{bmatrix} \tag{21}$$

which seems to give suitable starting values throughout the range of interest for conventional craft. It should be emphasized that Eq. (21) is not at all intended to serve as an approximate solution, but only to give a starting point from which the iteration will converge to the correct solution.

The numerical results presented in Fig. 3 have been calculated by this routine, and cover most of the range of parameters (η_s,η_h,κ) encountered in conventional craft, so they may be of some value in practical design. Contours of the objective $y=V_S\cos\gamma/V_T$ are shown as a function of η_h and κ for three different values of η_s. Interpolation in η_s may be carried out between the figures. Also the corresponding values of τ may be read from the graphs, and checked to see that allowable sail lift coefficients are not exceeded.

Throughout the range of parameters shown in Fig. 3, an increase in η_h or η_s, or a decrease in κ, improves windward performance as measured by y. Outside this range, y goes through a well-defined maximum with respect to κ, as shown in Fig. 4 for typical values of η_s and η_h. However, these optimum values of κ are an order of magnitude below those found in conventional craft, where, of course, light-wind performance is not the only design objective.

V. Limited Sail Lift

Milgram[11] discusses the maximum lift coefficients attainable without separation on cambered sails with zero thickness. He estimates section lift coefficients of 2.2 and overall lift coefficients $C_{L_{max}}$ of 1.7 (referred to total sail area A_s) are attainable with conventional planforms and properly shaped sails. The number comparable to this lift coefficient in present terms is $2C_s\tau/A_s$. If this exceeds $C_{L_{max}}$ when τ takes its optimum value, the optimum solution obtained is not physically realizable, and optimum windward performance in this case will instead be attained using τ fixed at $\tau_{max}=C_{L_{max}}A_s/2C_s$.

With τ fixed, the necessary conditions for an optimum equilibrium state are Eqs. (8) and (15). The equation for Newton-Raphson iteration analogous to Eq. (20) is

$$\begin{bmatrix} F_x & F_\beta \\ G_x & G_\beta \end{bmatrix} \begin{bmatrix} x_{n+1}-x_n \\ \beta_{n+1}-\beta_n \end{bmatrix} = -\begin{bmatrix} F \\ G \end{bmatrix} \tag{22}$$

In every case tested, the values of x and β that resulted from the iteration Eq. (20) with τ free served as satisfactory starting values for Eq. (22), and slightly lower values of y were found. Since the optimum performance in the case of limited lift depends on four parameters (η_s,η_h,κ, and τ_{max}), it is not practical to give general charts of the results.

VI. Numerical Example

To demonstrate application of the graphs and the iterative solutions, consider a concrete example.

Regression analysis of full-scale towing tests of the 5.5-m racing yacht *Antiope* led to a best-fit formula for the resistance data below 4 knots forward speed [14]

$$C_R=0.0108+C_S^2/0.59\pi$$

with both C_R and C_S (resistance and side-force coefficients, respectively) referred to the projected lateral area, $S_{ref}=53.3$ ft^2. This formula, recast in the form of Eq. (3), yields the parasitic and capture areas for the hull:

$$P_h=0.0108\cdot 53.3\ \text{ft}^2=0.577\ \text{ft}^2$$
$$C_h=(\pi/4)\cdot 0.59\cdot 53.3\ \text{ft}^2=24.7\ \text{ft}^2$$

A plausible rig for this boat, chosen from the cases calculated by Milgram,[12] is the "three-quarters" rig with both mainsail and jib aspect ratios (height/foot) of 3, a 50% jib overlap, and a mast height of 40 ft above deck. The total sail area is 362 ft^2. At a lift coefficient of 1.230, the induced drag coefficient calculated by vortex lattice theory is 0.153. From Eq. (4), the sail capture area is then $C_s=L^2/4QD=(1.23\cdot Q\cdot 362\ \text{ft}^2)^2/4\cdot Q(0.153\cdot Q\cdot 362\ \text{ft}^2)=896\ \text{ft}^2$. Assume that the windage of the hull, mast and rigging, and the skin friction on the sails, is equivalent to a parasitic drag area of 24.0 ft^2.

The dimensionless parameters of the boat sailing in sea water and standard sea-level air are:

$$\eta_s=37.33,\quad \eta_h=42.81,\quad \kappa=24.7\cdot 1.983/896\cdot .00238=22.97$$

Entering the three graphs of Fig. 3 with η_h and κ, we find these values

η_s	y	τ
20	0.584	0.422
30	0.595	0.410
40	0.618	0.398

Quadratic interpolation gives the solution $y=0.611$, $\tau=0.401$, for $\eta_s=37.33$.

For comparison, Eq. (21) gives starting values for (x_o,τ_o,β_o) of (1.871, 0.408, 0.455) and the iteration (20) converges in 3 steps to the values (1.884, 0.403, 0.445), giving $y=0.609$. Optimum performance occurs on a true course $\gamma=49.2°$ from the true wind (Eq. 12), with the boat traveling at a speed of 0.933 V_T (Eq. 11). The apparent wind is 1.76 times V_T, with dynamic pressure 3.09 times that of the true wind. This illustrates the degree to which an efficient vessel can "generate its own wind" in light air.

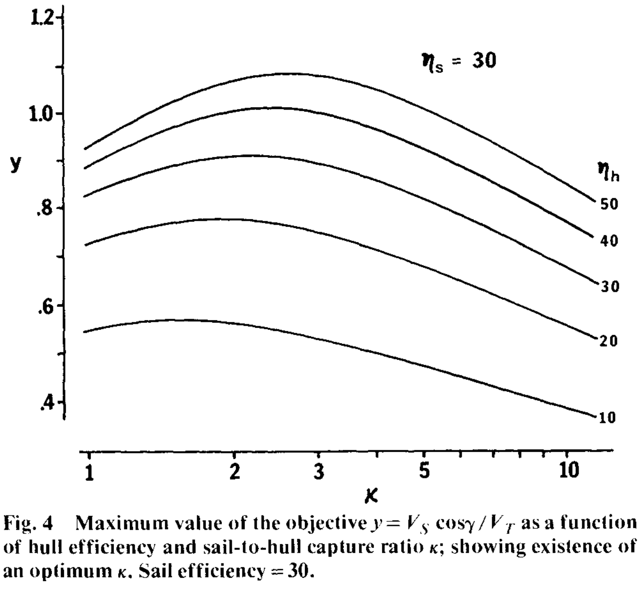

Fig. 4 Maximum value of the objective $y=V_S\cos\gamma/V_T$ as a function of hull efficiency and sail-to-hull capture ratio κ; showing existence of an optimum κ. Sail efficiency = 30.

The optimum solution with $\tau = 0.403$, however, calls for a sail lift coefficient of $2 \cdot 896 \cdot 0.403/362 = 1.99$, likely unattainable with conventional sails within this sail plan. Assuming $C_{L\max} = 1.7$ leads to $\tau_{\max} = 1.7 \cdot 362/2 \cdot 896 = 0.343$; and using starting values for (x_o, β_o) of (1.884, 0.445), the iteration (22) converges in 3 steps to the values (1.936, 0.441), giving $y = 0.602$, about 1% lower than the unconstrained optimum. This solution occurs at $\gamma = 47.7°$ from the true wind (Eq. 12).

VII. Conclusions

A significant range of operating conditions has been identified in which hull and sail characteristics are unaffected by wavemaking and heeling. In this regime, boat speed is directly proportional to true wind speed, and the vehicle can be described in an analytic model by three or four dimensionless parameters. Optimum sail trim is found to depend on hull parameters, as well as on characteristics of the sail, and vessel course and speed.

Conspicuous in their absence are considerations of lift-curve-slopes and angles of attack of sails or hull; these are found to have no direct bearing on performance, within the assumptions of the model. Lift-to-drag ratios of hull and sail do not appear as fundamental parameters, as is often assumed; and maximum lift-to-drag ratios are in general not optimum operating points for either hull or sails. An optimum proportion between hull and sails, as represented by the hull-to-sail capture ratio κ, is located, but reaching this optimum would require much larger sails than are ordinarily used.

Although the paper has been limited to a special range of operation in which simple force equations suffice, much of the present analytic approach and method of solution clearly can apply to a broader analysis of sailing performance. In particular, the treatment of sail forces and trim developed here is the first analytic alternative to use of the *Gimcrack* coefficients or similar empirical data in predicting sailing speeds from either model tests or theories of sailing hull performance.

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Announcement: 1976 Author and Subject Index

The indexes of the four AIAA archive journals (*AIAA Journal, Journal of Spacecraft and Rockets, Journal of Aircraft,* and *Journal of Hydronautics*) will be combined and mailed separately early in 1977. In addition, papers appearing in volumes of the *Progress in Astronautics and Aeronautics* book series published in 1977 will be included. Librarians will receive one copy of the index for each subscription which they have. Any AIAA member who subscribes to one or more Journals will receive one index. Additional copies may be purchased by anyone, at $10 per copy, from the Circulation Department, AIAA, Room 730, 1290 Avenue of the Americas, New York, New York 10019. **Remittance must accompany the order.**

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